CLASS 11 MATHEMATICS — FORMULA SHEET

Chapter 8 : Sequences and Series (NCERT)

8.1 Basic Definitions

Term	Definition
Sequence	An ordered list of numbers following a rule ; $a_1, a_2, a_3, \ldots$
Series	Sum of the terms of a sequence ; $a_1 + a_2 + a_3 + \ldots$
Finite sequence	Has a definite (countable) number of terms
Infinite sequence	Continues without end
General (nth) term	a_n — formula giving any term directly

8.2 Arithmetic Progression (AP)

Concept	Formula
AP form	$a, a+d, a+2d, \ldots$ (a = first term, d = common difference)
Common difference	$d = a_n - a_{(n-1)}$
nth term	$a_n = a + (n - 1)d$
nth term from end	$= l - (n - 1)d$ (l = last term)
Sum of n terms	$S_n = (n/2)[2a + (n - 1)d]$
Sum (first & last known)	$S_n = (n/2)(a + l)$
nth term from sum	$a_n = S_n - S_{(n-1)}$
Arithmetic mean (AM)	AM of a and $b = (a + b)/2$
n AMs between a, b	Common difference $d = (b - a)/(n + 1)$

8.3 Geometric Progression (GP)

Concept	Formula
GP form	$a, ar, ar^2, \ldots$ (a = first term, r = common ratio)
Common ratio	$r = a_n \div a_{(n-1)}$
nth term	$a_n = a\, r^{(n-1)}$
Sum of n terms (r ≠ 1)	$S_n = a(r^n - 1)/(r - 1) = a(1 - r^n)/(1 - r)$
Sum when r = 1	$S_n = na$
Sum to infinity (|r| < 1)	$S_\infty = a \div (1 - r)$
Geometric mean (GM)	GM of a and $b = \sqrt{(ab)}$
n GMs between a, b	Common ratio $r = (b/a)^{(1/(n+1))}$

8.4 Relation Between AM, GM (and HM)

Concept	Formula
Arithmetic mean	$A = (a + b)/2$
Geometric mean	$G = \sqrt{(ab)}$
Harmonic mean	$H = 2ab/(a + b)$
AM–GM inequality	$A \geq G \geq H$ (equal only when $a = b$)
Key relation	$G^2 = A \times H$ (G is the GM of A and H)

8.5 Sums of Special Series

Series	Sum Formula
First n natural numbers	$\Sigma n = n(n + 1)/2$
First n odd numbers	$1 + 3 + 5 + \ldots = n^2$
First n even numbers	$2 + 4 + 6 + \ldots = n(n + 1)$
Sum of squares	$\Sigma n^2 = n(n + 1)(2n + 1)/6$
Sum of cubes	$\Sigma n^3 = [n(n + 1)/2]^2 = (\Sigma n)^2$
Note	Sum of cubes = square of the sum of naturals

8.6 Useful AP/GP Selection Tricks

Number of terms	Convenient Way to Take Them
3 terms in AP	$a - d, a, a + d$
4 terms in AP	$a - 3d, a - d, a + d, a + 3d$ (common difference 2d)
5 terms in AP	$a - 2d, a - d, a, a + d, a + 2d$
3 terms in GP	$a/r, a, ar$
4 terms in GP	$a/r^3, a/r, ar, ar^3$ (common ratio r^2)
Why use these	Makes the sum/product symmetric and easy to solve

★ QUICK MEMORY AIDS & TRICKS

AP nth term	$a_n = a + (n-1)d$. Sum $S_n = (n/2)[2a+(n-1)d] = (n/2)(a+l)$.
GP nth term	$a_n = ar^{(n-1)\text{ ; Sum } = a(r^n-1)/(r-1)\text{ ; Infinite sum } = a/(1-r)\text{ when } \lvert r\rvert < 1.}$
AM-GM-HM	$A \geq G \geq H$ always ; and $G^2 = A \cdot H$.
Special sums	$\Sigma n = n(n+1)/2$; $\Sigma n^2 = n(n+1)(2n+1)/6$; $\Sigma n^3 = (\Sigma n)^2$.
Odd/even sums	First n odds = n^2 ; first n evens = $n(n+1)$.
3-term tricks	AP: $a-d, a, a+d$; GP: $a/r, a, ar$ — symmetry simplifies everything.
a_n from S_n	$a_n = S_n - S_{(n-1)}$ works for ANY series.
Mean insertion	AP: $d = (b-a)/(n+1)$; GP: $r = (b/a)^{(1/(n+1)).}$