CLASS 11 MATHEMATICS — FORMULA SHEET

Chapter 4 : Complex Numbers and Quadratic Equations (NCERT)

4.1 Complex Numbers — Basics

Concept	Definition / Formula
Imaginary unit	$i = \sqrt{(-1)}$; $i^2 = -1$
Complex number	$z = a + ib$; $a = \mathrm{Re}(z)$ (real part), $b = \mathrm{Im}(z)$ (imaginary part)
Purely real / imaginary	$b = 0 \to$ real ; $a = 0 \to$ purely imaginary
Equality	$a + ib = c + id \Leftrightarrow a = c$ and $b = d$
Powers of i	$i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$ (cycle repeats every 4)
i^n rule	Divide n by 4; use the remainder ($0\to1$, $1\to i$, $2\to-1$, $3\to-i$)

4.2 Algebra of Complex Numbers

Operation	Formula
Addition	$(a+ib) + (c+id) = (a+c) + i(b+d)$
Subtraction	$(a+ib) - (c+id) = (a-c) + i(b-d)$
Multiplication	$(a+ib)(c+id) = (ac - bd) + i(ad + bc)$
Conjugate of z	$\bar{z} = a - ib$ (changes sign of imaginary part)
Division	$z_1/z_2 = (z_1 \times \bar{z}_2) \div \lvert z_2\rvert^2$ (multiply by conjugate)
Reciprocal	$1/z = \bar{z} \div \lvert z\rvert^2 = (a - ib) \div (a^2 + b^2)$

4.3 Modulus & Conjugate Properties

Property	Formula
Modulus	$\lvert z\rvert = \sqrt{(a^2 + b^2)}$
Modulus of product	$\lvert z_1 z_2\rvert = \lvert z_1\rvert \lvert z_2\rvert$
Modulus of quotient	$\lvert z_1/z_2\rvert = \lvert z_1\rvert \div \lvert z_2\rvert$
z times conjugate	$z \cdot \bar{z} = \lvert z\rvert^2 = a^2 + b^2$
Conjugate of sum/product	$\overline{(z_1 \pm z_2)} = \bar{z}_1 \pm \bar{z}_2$; $\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$
Conjugate of conjugate	$\overline{(\bar{z})} = z$
Real & imaginary parts	$\mathrm{Re}(z) = (z + \bar{z})/2$; $\mathrm{Im}(z) = (z - \bar{z})/2i$
$\lvert z\rvert = \lvert\bar{z}\rvert = \lvert -z\rvert$	Modulus unaffected by conjugation or sign

4.4 Polar (Modulus–Argument) Form

Concept	Formula
Polar form	$z = r(\cos\theta + i \sin\theta)$, where $r = \lvert z\rvert$
Modulus r	$r = \sqrt{(a^2 + b^2)}$

Concept	Formula
Argument (amplitude) θ	$\tan\theta = b/a$; θ measured from positive real axis
Principal argument	$-\pi < \theta \le \pi$
Euler form	$z = r e^{i\theta}$ ($\cos\theta + i\sin\theta = e^{i\theta}$)
Multiplication (polar)	$z_1z_2 = r_1r_2 [\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$
Division (polar)	$z_1/z_2 = (r_1/r_2)[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)]$

4.5 Quadratic Equations & Roots

Concept	Formula
Standard form	$ax^2 + bx + c = 0$, $a \neq 0$
Quadratic formula	$x = [-b \pm \sqrt{(b^2 - 4ac)}] \div 2a$
Discriminant	$D = b^2 - 4ac$
Roots in complex case	If $D < 0$: $x = [-b \pm i\sqrt{(4ac - b^2)}] \div 2a$
Sum of roots	$\alpha + \beta = -b/a$
Product of roots	$\alpha\beta = c/a$
Form an equation from roots	$x^2 - (\text{sum})x + (\text{product}) = 0$

4.6 Nature of Roots (Discriminant Test)

Discriminant $D = b^2 - 4ac$	Nature of Roots
D > 0 (perfect square)	Real, rational, and distinct
D > 0 (not perfect square)	Real, irrational, distinct (conjugate surds)
D = 0	Real and equal (repeated root $x = -b/2a$)
D < 0	Imaginary (complex conjugate roots)
Complex roots occur in pairs	If $a+ib$ is a root, so is $a-ib$ (for real coefficients)

★ QUICK MEMORY AIDS & TRICKS

Powers of i	Cycle of 4: i, −1, −i, 1. Divide the power by 4 and use the remainder.
Division trick	To divide complex numbers, multiply top & bottom by the conjugate of the denominator.
$z \cdot \bar{z} = \lvert z\rvert^2$	Always real and non-negative. Handy for rationalising.
Modulus	$\lvert z\rvert = \sqrt{(a^2 + b^2)}$; $\lvert z\rvert = \lvert\bar{z}\rvert = \lvert -z\rvert$.
Sum & product	$\alpha+\beta = -b/a$, $\alpha\beta = c/a$. Build equation: $x^2 - (\text{sum})x + (\text{product}) = 0$.
Discriminant	D>0 real distinct, D=0 real equal, D<0 complex conjugate pair.
Complex roots	For real coefficients, complex roots ALWAYS come in conjugate pairs.
Polar multiply	Multiply moduli, ADD arguments ; divide moduli, SUBTRACT arguments.