

CLASS 12 MATHEMATICS — FORMULA SHEET

Chapter 7 : Integrals (NCERT)

7.1 Standard Integrals (add + C)

Function	Integral
$\int x^n dx$	$x^{(n+1)}/(n+1)$, $n \neq -1$
$\int 1/x dx$	$\ln x $
$\int e^x dx$	e^x
$\int a^x dx$	$a^x / \ln a$
$\int \sin x dx$	$-\cos x$
$\int \cos x dx$	$\sin x$
$\int \sec^2 x dx$	$\tan x$
$\int \operatorname{cosec}^2 x dx$	$-\cot x$
$\int \sec x \tan x dx$	$\sec x$
$\int \operatorname{cosec} x \cot x dx$	$-\operatorname{cosec} x$
$\int \tan x dx$	$\ln \sec x = -\ln \cos x $
$\int \cot x dx$	$\ln \sin x $
$\int \sec x dx$	$\ln \sec x + \tan x $
$\int \operatorname{cosec} x dx$	$\ln \operatorname{cosec} x - \cot x $

7.2 Special Integrals (Inverse Trig & Log Forms)

Integral	Result (+ C)
$\int 1/\sqrt{1-x^2} dx$	$\sin^{-1} x$
$\int 1/(1+x^2) dx$	$\tan^{-1} x$
$\int 1/(x\sqrt{x^2-1}) dx$	$\sec^{-1} x$
$\int 1/(a^2 + x^2) dx$	$(1/a) \tan^{-1} (x/a)$
$\int 1/(x^2 - a^2) dx$	$(1/2a) \ln (x-a)/(x+a) $
$\int 1/(a^2 - x^2) dx$	$(1/2a) \ln (a+x)/(a-x) $
$\int 1/\sqrt{a^2 - x^2} dx$	$\sin^{-1} (x/a)$
$\int 1/\sqrt{x^2 + a^2} dx$	$\ln x + \sqrt{x^2 + a^2} $
$\int 1/\sqrt{x^2 - a^2} dx$	$\ln x + \sqrt{x^2 - a^2} $

7.3 Methods of Integration

Method	Detail
Substitution	Let $u = g(x)$; convert the integral in terms of u
$\int f(g(x))g'(x) dx$	$= \int f(u) du$ with $u = g(x)$

Method	Detail
Integration by parts	$\int u v \, dx = u \int v \, dx - \int (u' \int v \, dx) \, dx$
ILATE rule	Choose u in order: Inverse, Log, Algebraic, Trig, Exp
Partial fractions	Break a rational function into simpler fractions
$\int e^x[f(x)+f'(x)] \, dx$	$= e^x f(x) + C$

7.4 Special Forms (Integration by Parts results)

Integral	Result (+ C)
$\int \sqrt{a^2 - x^2} \, dx$	$(x/2)\sqrt{a^2 - x^2} + (a^2/2) \sin^{-1}(x/a)$
$\int \sqrt{x^2 + a^2} \, dx$	$(x/2)\sqrt{x^2 + a^2} + (a^2/2) \ln x + \sqrt{x^2 + a^2} $
$\int \sqrt{x^2 - a^2} \, dx$	$(x/2)\sqrt{x^2 - a^2} - (a^2/2) \ln x + \sqrt{x^2 - a^2} $

7.5 Definite Integrals & Properties

Property	Formula
Fundamental theorem	$\int(a \text{ to } b) f(x) \, dx = F(b) - F(a)$, $F'(x)=f(x)$
Reverse limits	$\int(a \text{ to } b) f = -\int(b \text{ to } a) f$
Same limits	$\int(a \text{ to } a) f = 0$
King's property	$\int(a \text{ to } b) f(x) \, dx = \int(a \text{ to } b) f(a+b-x) \, dx$
Zero to a	$\int(0 \text{ to } a) f(x) \, dx = \int(0 \text{ to } a) f(a-x) \, dx$
Even function	$\int(-a \text{ to } a) f = 2\int(0 \text{ to } a) f$, if f even
Odd function	$\int(-a \text{ to } a) f = 0$, if f odd
Split interval	$\int(a \text{ to } b) = \int(a \text{ to } c) + \int(c \text{ to } b)$

★ QUICK MEMORY AIDS & TRICKS

Always + C	Indefinite integrals need the constant of integration C.
ILATE for parts	Pick u as Inverse > Log > Algebraic > Trig > Exp (in that priority).
e^x trick	$\int e^x[f(x)+f'(x)]dx = e^x f(x)$. Spot the function plus its derivative.
tan/sec integrals	$\int \tan x = \ln \sec x $; $\int \sec x = \ln \sec x + \tan x $ (memorise these).
$a^2 \pm x^2$ forms	$1/(a^2+x^2) \rightarrow (1/a)\tan^{-1}(x/a)$; $1/\sqrt{a^2-x^2} \rightarrow \sin^{-1}(x/a)$.
Partial fractions	Use when integrating proper rational functions (factor the denominator).
King's property	$\int_0^a f(x)dx = \int_0^a f(a-x)dx$ — powerful for definite integrals.
Even/odd	Over $[-a, a]$: even $\rightarrow 2\int_0^a$; odd $\rightarrow 0$. Check symmetry first.