CLASS 12 MATHEMATICS — FORMULA SHEET

Chapter 5 : Continuity and Differentiability (NCERT)

5.1 Continuity

Concept	Detail
Continuous at x = a	LHL = RHL = f(a) ; i.e. $\lim(x \to a) f(x) = f(a)$
Left-hand limit	$\lim(x \to a^-) f(x)$
Right-hand limit	$\lim(x \to a^+) f(x)$
Discontinuous if	Any of the three values differ or don't exist
Sum/diff/product	Continuous functions stay continuous under these
Quotient f/g	Continuous where $g \neq 0$
Composite	If f & g continuous, $g \circ f$ is continuous

5.2 Differentiability

Concept	Detail
Differentiable at x = a	LHD = RHD (left & right derivatives equal)
First principle	$f'(x) = \lim(h \to 0)[f(x+h) - f(x)] \div h$
Key theorem	Differentiable ⇒ continuous (converse not always true)
Example (not differentiable)	$f(x) = \|x\|$ at $x = 0$ (continuous but sharp corner)
LHD	$\lim(h \to 0^-)[f(a+h) - f(a)] \div h$
RHD	$\lim(h \to 0^+)[f(a+h) - f(a)] \div h$

5.3 Standard Derivatives

Function	Derivative
x^n	$n\,x^{(n-1)}$
sin x / cos x	cos x / −sin x
tan x / cot x	$\sec^2 x$ / $-\text{cosec}^2 x$
sec x / cosec x	sec x tan x / −cosec x cot x
e^x / a^x	e^x / $a^x \log_e a$
$\log_e x$ / $\log_a x$	$1/x$ / $1/(x \log_e a)$
$\sin^{-1} x$ / $\cos^{-1} x$	$1/\sqrt{(1-x^2)}$ / $-1/\sqrt{(1-x^2)}$
$\tan^{-1} x$ / $\cot^{-1} x$	$1/(1+x^2)$ / $-1/(1+x^2)$
$\sec^{-1} x$ / $\text{cosec}^{-1} x$	$1/(\|x\|\sqrt{(x^2-1)})$ / $-1/(\|x\|\sqrt{(x^2-1)})$

5.4 Rules of Differentiation

Rule	Formula
Sum/difference	$(u \pm v)' = u' \pm v'$
Product rule	$(uv)' = u'v + uv'$
Quotient rule	$(u/v)' = (u'v - uv') \div v^2$
Chain rule	$d/dx\, f(g(x)) = f'(g(x)) \cdot g'(x)$
Implicit differentiation	Differentiate both sides w.r.t. x, solve for dy/dx
Parametric form	$dy/dx = (dy/dt) \div (dx/dt)$
Logarithmic differentiation	For $y = [f(x)]^{g(x)}$: take log, then differentiate

5.5 Special Differentiation & Second Derivative

Concept	Detail / Formula
$y = a^x$ form (use log)	$\ln y = x \ln a$; $(1/y)(dy/dx) = \ln a$
$y = x^x$	$dy/dx = x^{x\,(1 + \ln x)}$
Second derivative	$d^2y/dx^2 = d/dx\,(dy/dx)$
Parametric 2nd derivative	Differentiate dy/dx w.r.t. t, then divide by dx/dt
Higher derivatives	Differentiate repeatedly

5.6 Mean Value Theorems

Theorem	Statement
Rolle's theorem	If f continuous on [a,b], differentiable on (a,b), and f(a)=f(b), then f'(c)=0 for some c
Lagrange's MVT	$f'(c) = [f(b) - f(a)] \div (b - a)$ for some c in (a,b)
Geometric meaning (MVT)	Tangent at c is parallel to the chord joining endpoints
Conditions needed	Continuity on [a,b] and differentiability on (a,b)

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Continuity test	LHL = RHL = f(a). All three must match for continuity at a point.
Differentiable ⇒ continuous	True one way only. |x| is continuous at 0 but NOT differentiable.
Inverse trig derivatives	$\sin^{-1} \to 1/\sqrt{(1-x^2)}$; $\tan^{-1} \to 1/(1+x^2)$. Co-functions get a minus sign.
Chain rule	Differentiate outer, keep inner, times derivative of inner.
Log differentiation	Use for x^x, variable powers, and big products/quotients.
Parametric	dy/dx = (dy/dt)/(dx/dt). Don't forget to divide again for the 2nd derivative.
Rolle vs MVT	Rolle needs f(a)=f(b) and gives f'(c)=0 ; MVT is the general slope version.
x^x derivative	$d/dx(x^x) = x^x(1 + \ln x)$. A classic log-differentiation result.