CLASS 12 MATHEMATICS — FORMULA SHEET

Chapter 4 : Determinants (NCERT)

4.1 Determinant — Definition

Concept	Formula
Determinant (2×2)	$\lvert A\rvert = a_{11}a_{22} - a_{12}a_{21}$
Determinant (3×3)	Expand along any row/column using cofactors
Notation	$\det A = \lvert A\rvert = \Delta$
Defined only for	Square matrices
Singular matrix	$\lvert A\rvert = 0$
Non-singular matrix	$\lvert A\rvert \neq 0$ (invertible)

4.2 Properties of Determinants

Property	Detail
Row–column swap	$\lvert A'\rvert = \lvert A\rvert$ (transpose leaves determinant unchanged)
Interchange two rows	Sign of determinant changes
Two identical rows/cols	Determinant = 0
Multiply a row by k	Determinant becomes $k\lvert A\rvert$
Common factor	k can be taken out of one row/column
Row addition	Adding a multiple of one row to another → no change
Triangular matrix	Determinant = product of diagonal elements
$\lvert kA\rvert$ (n×n)	$= k^n \lvert A\rvert$
$\lvert AB\rvert$	$= \lvert A\rvert \times \lvert B\rvert$

4.3 Area of Triangle & Minors/Cofactors

Concept	Formula
Area of triangle	$\Delta = \frac{1}{2} \lvert x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\rvert$
Collinearity condition	Area = 0 (determinant of points = 0)
Minor M_{ij}	Determinant after deleting i-th row & j-th column
Cofactor A_{ij}	$A_{ij} = (-1)^{(i+j)} M_{ij}$
Expansion along a row	$\lvert A\rvert = a_{i1}A_{i1} + a_{i2}A_{i2} + a_{i3}A_{i3}$
Sum of products (wrong row)	= 0 (element × cofactor of another row)

4.4 Adjoint & Inverse

Concept	Formula
Adjoint	adj A = transpose of the cofactor matrix

Concept	Formula
A (adj A)	= (adj A) A = $\|A\|$ I
Inverse	A^{-1} = (adj A) ÷ $\|A\|$, provided $\|A\| \neq 0$
$\|adj\ A\|$ (n×n)	= $\|A\|^{(n-1)}$
adj(AB)	= (adj B)(adj A)
$\|A^{-1}\|$	= 1 ÷ $\|A\|$
$(A^{-1})^{-1}$	= A
A is invertible ⇔	$\|A\| \neq 0$ (non-singular)

4.5 Solving Linear Equations

Concept	Detail
Matrix form	AX = B (A = coefficients, X = variables, B = constants)
Solution (matrix method)	$X = A^{-1} B$ (if $\|A\| \neq 0$)
Cramer's rule	$x = \Delta_1/\Delta$, $y = \Delta_2/\Delta$, $z = \Delta_3/\Delta$
Δ_i	Δ with i-th column replaced by constants B
Unique solution	If $\|A\| \neq 0$ (consistent, independent)
No solution	If $\|A\| = 0$ and (adj A)B ≠ 0 (inconsistent)
Infinite solutions	If $\|A\| = 0$ and (adj A)B = 0 (dependent)

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2×2 determinant	$\|A\|$ = ad − bc (main diagonal minus off diagonal).
Zero determinant	Two equal rows/columns → det = 0. Such a matrix is singular (no inverse).
Row operations	Swapping rows flips sign ; adding a multiple of a row changes nothing.
Cofactor sign	$A_{ij} = (-1)^{(i+j)} M_{ij}$. Sign chessboard: + − + / − + − / + − +.
Inverse formula	A^{-1} = adj A / $\|A\|$. Needs $\|A\| \neq 0$.
$\|adj\ A\|$	= $\|A\|^{(n-1)}$ for an n×n matrix.
Cramer's rule	$x_i = \Delta_i/\Delta$ — replace the i-th column with constants.
Consistency	$\|A\| \neq 0$ unique ; $\|A\|=0$ → either no solution or infinitely many.