

CLASS 12 MATHEMATICS — FORMULA SHEET

Chapter 3 : Matrices (NCERT)

3.1 Basics & Order of a Matrix

Concept	Detail
Matrix	Rectangular array of numbers arranged in rows & columns
Order	$m \times n$ (m rows, n columns)
Element a_{ij}	Entry in i-th row, j-th column
Number of elements	$m \times n$
Equal matrices	Same order AND corresponding elements equal
Number of $m \times n$ matrices	Possible with given entries set

3.2 Types of Matrices

Type	Definition
Row matrix	Single row ($1 \times n$)
Column matrix	Single column ($m \times 1$)
Square matrix	Rows = columns ($n \times n$)
Diagonal matrix	Non-diagonal elements all zero
Scalar matrix	Diagonal matrix with equal diagonal elements
Identity matrix (I)	Scalar matrix with diagonal elements = 1
Zero matrix	All elements zero
Upper/lower triangular	Zeros below/above the main diagonal

3.3 Matrix Operations

Operation	Rule
Addition	Add corresponding elements (same order required)
Scalar multiplication	Multiply every element by the scalar k
Multiplication condition	$A(m \times n) \times B(n \times p)$ defined only if columns of A = rows of B
Product order	Result is ($m \times p$)
$AB \neq BA$	Matrix multiplication is NOT commutative (in general)
Associative	$(AB)C = A(BC)$
Distributive	$A(B + C) = AB + AC$
Identity property	$AI = IA = A$

3.4 Transpose of a Matrix

Property	Formula
Transpose A'	Rows become columns (interchange)
$(A')'$	$= A$
$(A + B)'$	$= A' + B'$
$(kA)'$	$= k A'$
$(AB)'$	$= B' A'$ (order reverses)
Symmetric matrix	$A' = A$ ($a_{ij} = a_{ji}$)
Skew-symmetric matrix	$A' = -A$ (diagonal elements = 0)

3.5 Symmetric & Skew-Symmetric

Concept	Detail
Symmetric	$A' = A$
Skew-symmetric	$A' = -A$; main diagonal all zeros
Any square matrix	$A = (\text{symmetric part}) + (\text{skew-symmetric part})$
Symmetric part	$\frac{1}{2}(A + A')$
Skew-symmetric part	$\frac{1}{2}(A - A')$
$A + A'$	Always symmetric
$A - A'$	Always skew-symmetric

3.6 Elementary Operations & Invertible Matrices

Concept	Detail
Elementary row operations	Swap rows, scale a row, add a multiple of one row to another
Invertible matrix	A is invertible if there exists B with $AB = BA = I$
Inverse notation	$B = A^{-1}$
Inverse uniqueness	Inverse of a matrix, if it exists, is unique
$(AB)^{-1}$	$= B^{-1} A^{-1}$ (order reverses)
Inverse by elementary ops	Convert A to I via row ops applied also to I
Condition to be invertible	Square matrix with non-zero determinant (covered in Ch 4)

★ QUICK MEMORY AIDS & TRICKS

Multiplication rule	AB defined only when columns of A = rows of B . Result is $\text{rows}(A) \times \text{cols}(B)$.
Not commutative	$AB \neq BA$ in general — order always matters with matrices.
Transpose reverses	$(AB)' = B'A'$ and $(AB)^{-1} = B^{-1}A^{-1}$ — order flips.
Symmetric vs skew	Symmetric: $A' = A$. Skew: $A' = -A$ (diagonal all zeros).
Decomposition	Any square matrix $= \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$ (symmetric + skew parts).

Identity	$AI = IA = A$. The identity matrix acts like '1' for matrices.
Zero product trap	$AB = 0$ does NOT mean $A = 0$ or $B = 0$ (unlike numbers).
Inverse needs square	Only square matrices can have inverses ; and only if $\det \neq 0$.