CLASS 12 MATHEMATICS — FORMULA SHEET

Chapter 2 : Inverse Trigonometric Functions (NCERT)

2.1 Domains & Principal Value Ranges

Function	Domain	Principal Value Range
$\sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R (all reals)	$(-\pi/2, \pi/2)$
$\cot^{-1} x$	R (all reals)	$(0, \pi)$
$\sec^{-1} x$	$\lvert x\rvert \geq 1$	$[0, \pi] - \{\pi/2\}$
$\operatorname{cosec}^{-1} x$	$\lvert x\rvert \geq 1$	$[-\pi/2, \pi/2] - \{0\}$

2.2 Basic Properties (Self-Inverse)

Property	Condition
$\sin(\sin^{-1} x) = x$	$x \in [-1, 1]$
$\sin^{-1}(\sin x) = x$	$x \in [-\pi/2, \pi/2]$
$\cos(\cos^{-1} x) = x$	$x \in [-1, 1]$
$\tan(\tan^{-1} x) = x$	$x \in R$
$\sin^{-1}(-x) = -\sin^{-1} x$	Odd function
$\tan^{-1}(-x) = -\tan^{-1} x$	Odd function
$\cos^{-1}(-x) = \pi - \cos^{-1} x$	Not odd
$\cot^{-1}(-x) = \pi - \cot^{-1} x$	Not odd

2.3 Complementary & Reciprocal Relations

Identity	Condition
$\sin^{-1} x + \cos^{-1} x = \pi/2$	$x \in [-1, 1]$
$\tan^{-1} x + \cot^{-1} x = \pi/2$	$x \in R$
$\sec^{-1} x + \operatorname{cosec}^{-1} x = \pi/2$	$\lvert x\rvert \geq 1$
$\sin^{-1}(1/x) = \operatorname{cosec}^{-1} x$	$\lvert x\rvert \geq 1$
$\cos^{-1}(1/x) = \sec^{-1} x$	$\lvert x\rvert \geq 1$
$\tan^{-1}(1/x) = \cot^{-1} x$	$x > 0$

2.4 Sum & Difference Formulas

Identity	Condition
$\tan^{-1} x + \tan^{-1} y = \tan^{-1}[(x+y)/(1-xy)]$	$xy < 1$
$\tan^{-1} x - \tan^{-1} y = \tan^{-1}[(x-y)/(1+xy)]$	$xy > -1$

Identity	Condition
$2\tan^{-1}x = \tan^{-1}[2x/(1-x^2)]$	$\lvert x\rvert < 1$
$2\tan^{-1}x = \sin^{-1}[2x/(1+x^2)]$	$\lvert x\rvert \le 1$
$2\tan^{-1}x = \cos^{-1}[(1-x^2)/(1+x^2)]$	$x \ge 0$
$2\sin^{-1}x = \sin^{-1}[2x\sqrt{(1-x^2)}]$	$-1/\sqrt{2} \le x \le 1/\sqrt{2}$

2.5 Useful Conversions ($\sin^{-1}$ in terms of others)

Identity	Equivalent Forms
$\sin^{-1}x =$	$\cos^{-1}\sqrt{(1-x^2)} = \tan^{-1}[x/\sqrt{(1-x^2)}]$
$\cos^{-1}x =$	$\sin^{-1}\sqrt{(1-x^2)} = \tan^{-1}[\sqrt{(1-x^2)}/x]$
$\tan^{-1}x =$	$\sin^{-1}[x/\sqrt{(1+x^2)}] = \cos^{-1}[1/\sqrt{(1+x^2)}]$
Useful identity	$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$
Useful identity	$3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$
Useful identity	$3\tan^{-1}x = \tan^{-1}[(3x - x^3)/(1 - 3x^2)]$

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Principal ranges	$\sin^{-1}$/$\tan^{-1}$/$\operatorname{cosec}^{-1}$ → $[-\pi/2,\pi/2]$ family ; $\cos^{-1}$/$\cot^{-1}$/$\sec^{-1}$ → $[0,\pi]$ family.
Complementary pairs	Each co-pair sums to $\pi/2$: $\sin^{-1}+\cos^{-1}$, $\tan^{-1}+\cot^{-1}$, $\sec^{-1}+\operatorname{cosec}^{-1}$.
Odd functions	$\sin^{-1}$, $\tan^{-1}$, $\operatorname{cosec}^{-1}$ are odd: $f(-x) = -f(x)$.
$\cos^{-1}(-x)$	$= \pi - \cos^{-1}x$ (NOT odd). Same pattern for $\cot^{-1}$ and $\sec^{-1}$.
$\tan^{-1}$ sum	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}[(x+y)/(1-xy)]$ — watch the $xy<1$ condition.
Reciprocal rule	$\sin^{-1}(1/x) = \operatorname{cosec}^{-1}x$, etc. Reciprocal inside ↔ co/reciprocal function.
Always check range	$\sin^{-1}(\sin x) = x$ ONLY if x is in the principal range; else adjust.
Triangle trick	For mixed conversions, draw a right triangle with the given ratio.