CLASS 12 MATHEMATICS — FORMULA SHEET

Chapter 1 : Relations and Functions (NCERT)

1.1 Types of Relations

Relation	Condition (on set A)
Empty relation	No element related ; $R = \varphi$
Universal relation	Every element related ; $R = A \times A$
Reflexive	$(a, a) \in R$ for all $a \in A$
Symmetric	$(a, b) \in R \Rightarrow (b, a) \in R$
Transitive	$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
Equivalence relation	Reflexive + Symmetric + Transitive (all three)
Equivalence class [a]	Set of all elements related to a

1.2 Types of Functions

Function	Condition
One-one (injective)	$f(a) = f(b) \Rightarrow a = b$ (distinct inputs → distinct outputs)
Many-one	Two or more inputs give the same output
Onto (surjective)	Every element of codomain has a pre-image (range = codomain)
Into	At least one codomain element has no pre-image
Bijective	Both one-one AND onto
Number of bijections	From a set of n elements to itself = $n!$

1.3 Checking One-One & Onto

Method	Detail
One-one (algebraic)	Assume $f(x_1) = f(x_2)$ and prove $x_1 = x_2$
One-one (graphical)	Horizontal line cuts graph at most once
One-one (calculus)	$f'(x) > 0$ (strictly increasing) or $f'(x) < 0$ (strictly decreasing)
Onto	Show range = codomain (every y has some x)
Many-one example	$f(x) = x^2$ on R (since $f(-2) = f(2)$)

1.4 Composition of Functions

Concept	Formula
Composition	$(g \circ f)(x) = g(f(x))$
Order matters	$g \circ f \neq f \circ g$ (in general)
Associative	$(h \circ g) \circ f = h \circ (g \circ f)$
Composition of one-one	If f and g are one-one, then $g \circ f$ is one-one

Concept	Formula
Composition of onto	If f and g are onto, then $g \circ f$ is onto
Identity function	$I(x) = x$; $f \circ I = I \circ f = f$

1.5 Invertible Functions

Concept	Detail
Invertible function	A function f is invertible ⇔ f is bijective (one-one & onto)
Inverse definition	$f^{-1}(y) = x \Leftrightarrow f(x) = y$
Property	$f \circ f^{-1} = f^{-1} \circ f$ = identity function
Inverse of composition	$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ (order reverses)
Finding inverse	Write $y = f(x)$, solve for x, then swap → $f^{-1}(x)$
Inverse is unique	Each bijective function has exactly one inverse

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Equivalence relation	Must be Reflexive + Symmetric + Transitive — ALL three together.
One-one test	Set $f(x_1) = f(x_2)$; if it forces $x_1 = x_2$, the function is one-one.
Onto test	Range = codomain. Every output value must be achievable.
Bijective = invertible	A function has an inverse if and only if it is one-one AND onto.
Composition order	$(g\circ f)(x) = g(f(x))$ — apply f FIRST, then g. Order matters.
Inverse of composite	$(g\circ f)^{-1} = f^{-1}\circ g^{-1}$ — the order REVERSES (like socks & shoes).
Calculus shortcut	$f'(x)$ always >0 or always <0 → strictly monotonic → one-one.
Bijections count	Number of bijections from an n-set to itself = $n!$.