CLASS 12 MATHEMATICS — FORMULA SHEET

Chapter 11 : Three Dimensional Geometry (NCERT)

11.1 Direction Cosines & Direction Ratios

Concept	Formula
Direction cosines (DCs)	$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$
Property of DCs	$l^2 + m^2 + n^2 = 1$
Direction ratios (DRs)	a, b, c proportional to l, m, n
DCs from DRs	$l = a/\sqrt{(a^2+b^2+c^2)}$, and similarly for m, n
DRs from two points	$(x_2-x_1,\ y_2-y_1,\ z_2-z_1)$
Distance between points	$\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2]}$

11.2 Equation of a Line

Form	Equation
Vector form (point + direction)	$r = a + \lambda b$
Cartesian (symmetric) form	$(x-x_1)/a = (y-y_1)/b = (z-z_1)/c$
Through two points (vector)	$r = a + \lambda(b - a)$
Through two points (Cartesian)	$(x-x_1)/(x_2-x_1) = (y-y_1)/(y_2-y_1) = (z-z_1)/(z_2-z_1)$
a, b, c here are	Direction ratios of the line

11.3 Angle Between Two Lines

Concept	Formula
Angle (direction ratios)	$\cos\theta = \lvert a_1a_2 + b_1b_2 + c_1c_2\rvert \div (\sqrt{(a_1^2+b_1^2+c_1^2)}\ \sqrt{(a_2^2+b_2^2+c_2^2)})$
Angle (vector form)	$\cos\theta = \lvert b_1 \cdot b_2\rvert \div (\lvert b_1\rvert\lvert b_2\rvert)$
Parallel lines	$a_1/a_2 = b_1/b_2 = c_1/c_2$
Perpendicular lines	$a_1a_2 + b_1b_2 + c_1c_2 = 0$

11.4 Equation of a Plane

Form	Equation
General Cartesian form	$Ax + By + Cz + D = 0$
Normal vector to plane	(A, B, C) are direction ratios of the normal
Vector form (normal n)	$r \cdot \hat{n} = d$ (d = distance from origin)
Point + normal form	$A(x-x_1) + B(y-y_1) + C(z-z_1) = 0$
Intercept form	$x/a + y/b + z/c = 1$
Plane through 3 points	Use determinant of coordinates = 0

11.5 Angles & Distances

Concept	Formula
Angle between two planes	$\cos\theta = \lvert A_1A_2+B_1B_2+C_1C_2\rvert \div (\sqrt{(\Sigma A_1^2)}\ \sqrt{(\Sigma A_2^2)})$
Parallel planes	$A_1/A_2 = B_1/B_2 = C_1/C_2$
Perpendicular planes	$A_1A_2 + B_1B_2 + C_1C_2 = 0$
Angle between line & plane	$\sin\theta = \lvert a\cdot n\rvert \div (\lvert a\rvert\lvert n\rvert)$
Distance: point to plane	$\lvert Ax_1+By_1+Cz_1+D\rvert \div \sqrt{(A^2+B^2+C^2)}$
Line parallel to plane	$a\cdot n = 0$ (direction $\perp$ normal)

★ QUICK MEMORY AIDS & TRICKS

Direction cosines	$l^2+m^2+n^2 = 1$ always. DCs are the cosines of angles with the axes.
Line equation	Vector: $r = a + \lambda b$ (point a, direction b). Cartesian uses the same DRs.
Parallel vs perpendicular	Parallel: DRs proportional. Perpendicular: $a_1a_2+b_1b_2+c_1c_2 = 0$.
Plane normal	In Ax+By+Cz+D=0, the coefficients (A,B,C) form the normal vector.
Angle line–plane	Uses SINE ($\sin\theta = \lvert a\cdot n\rvert/\lvert a\rvert\lvert n\rvert$) — note: with the NORMAL.
Angle line–line / plane–plane	Uses COSINE of the direction/normal vectors.
Point–plane distance	$\lvert Ax_1+By_1+Cz_1+D\rvert / \sqrt{(A^2+B^2+C^2)}$. Plug the point into the plane.
Intercept form	$x/a + y/b + z/c = 1$ gives axis intercepts a, b, c directly.