

# CLASS 12 MATHEMATICS — FORMULA SHEET

Chapter 10 : Vector Algebra (NCERT)

## 10.1 Basics & Types of Vectors

| Concept                                                    | Detail                                                                 |
|------------------------------------------------------------|------------------------------------------------------------------------|
| Vector                                                     | Quantity with magnitude AND direction                                  |
| Magnitude of $\mathbf{a} = x\hat{i} + y\hat{j} + z\hat{k}$ | $ \mathbf{a}  = \sqrt{x^2 + y^2 + z^2}$                                |
| Position vector of P(x,y,z)                                | $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$                          |
| Zero vector                                                | Magnitude 0, no fixed direction                                        |
| Unit vector                                                | $\hat{\mathbf{a}} = \mathbf{a} /  \mathbf{a} $ (magnitude 1)           |
| Equal vectors                                              | Same magnitude AND same direction                                      |
| Collinear vectors                                          | Parallel ; $\mathbf{a} = \lambda \mathbf{b}$ for some scalar $\lambda$ |
| Coplanar vectors                                           | Lie in the same plane                                                  |

## 10.2 Vector Operations & Section Formula

| Concept                           | Formula                                                      |
|-----------------------------------|--------------------------------------------------------------|
| Addition (triangle/parallelogram) | Resultant of two vectors                                     |
| Scalar multiplication             | $\lambda \mathbf{a}$ scales magnitude by $ \lambda $         |
| Direction cosines                 | $l = x/ \mathbf{r} , m = y/ \mathbf{r} , n = z/ \mathbf{r} $ |
| Property of DCs                   | $l^2 + m^2 + n^2 = 1$                                        |
| Section formula (internal)        | $\mathbf{r} = (m \mathbf{b} + n \mathbf{a}) \div (m + n)$    |
| Section formula (external)        | $\mathbf{r} = (m \mathbf{b} - n \mathbf{a}) \div (m - n)$    |
| Midpoint                          | $\mathbf{r} = (\mathbf{a} + \mathbf{b}) \div 2$              |

## 10.3 Dot (Scalar) Product

| Concept                                    | Formula                                                                       |
|--------------------------------------------|-------------------------------------------------------------------------------|
| Definition                                 | $\mathbf{a} \cdot \mathbf{b} =  \mathbf{a}  \mathbf{b}  \cos \theta$          |
| Component form                             | $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$                      |
| Angle between vectors                      | $\cos \theta = (\mathbf{a} \cdot \mathbf{b}) \div ( \mathbf{a}  \mathbf{b} )$ |
| Perpendicular vectors                      | $\mathbf{a} \cdot \mathbf{b} = 0$                                             |
| Magnitude squared                          | $\mathbf{a} \cdot \mathbf{a} =  \mathbf{a} ^2$                                |
| Projection of $\mathbf{a}$ on $\mathbf{b}$ | $(\mathbf{a} \cdot \mathbf{b}) \div  \mathbf{b} $                             |
| Commutative                                | $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$                   |
| Result type                                | Scalar (a number)                                                             |

## 10.4 Cross (Vector) Product

| Concept               | Formula                                                                                   |
|-----------------------|-------------------------------------------------------------------------------------------|
| Definition            | $a \times b =  a  b  \sin \theta \hat{n}$                                                 |
| Magnitude             | $ a \times b  =  a  b  \sin \theta$                                                       |
| Component form        | $a \times b = \text{determinant of } [\hat{i} \hat{j} \hat{k}; a_1 a_2 a_3; b_1 b_2 b_3]$ |
| Parallel vectors      | $a \times b = 0$                                                                          |
| Anti-commutative      | $a \times b = -(b \times a)$                                                              |
| Area of parallelogram | $ a \times b $ (adjacent sides a, b)                                                      |
| Area of triangle      | $\frac{1}{2}  a \times b $                                                                |
| Unit normal vector    | $\hat{n} = (a \times b) \div  a \times b $                                                |
| Result type           | Vector (perpendicular to both)                                                            |

## 10.5 Dot vs Cross — Comparison

| Feature      | Dot Product                           | Cross Product                     |
|--------------|---------------------------------------|-----------------------------------|
| Result       | Scalar                                | Vector                            |
| Formula      | $ a  b  \cos \theta$                  | $ a  b  \sin \theta \hat{n}$      |
| Zero when    | Perpendicular ( $\theta = 90^\circ$ ) | Parallel ( $\theta = 0^\circ$ )   |
| Commutative? | Yes ( $a \cdot b = b \cdot a$ )       | No ( $a \times b = -b \times a$ ) |
| Main use     | Angle, projection                     | Area, normal direction            |

## ★ QUICK MEMORY AIDS & TRICKS

|                     |                                                                                              |
|---------------------|----------------------------------------------------------------------------------------------|
| Magnitude           | $ a  = \sqrt{x^2+y^2+z^2}$ . Unit vector = $a/ a $ .                                         |
| Dot = scalar        | $a \cdot b =  a  b \cos\theta$ . Zero when perpendicular. Gives the angle & projection.      |
| Cross = vector      | $a \times b =  a  b \sin\theta \hat{n}$ . Zero when parallel. Gives area & normal direction. |
| cos vs sin          | Dot uses COSine (cos→perpendicular zero) ; cross uses SIne (sin→parallel zero).              |
| Areas               | Parallelogram = $ a \times b $ ; triangle = $\frac{1}{2} a \times b $ .                      |
| Direction cosines   | $l^2+m^2+n^2 = 1$ always.                                                                    |
| Section formula     | Internal: $(mb+na)/(m+n)$ . Midpoint: $(a+b)/2$ .                                            |
| Cross order matters | $a \times b = -(b \times a)$ . Dot order doesn't: $a \cdot b = b \cdot a$ .                  |