CLASS 12 MATHEMATICS — FORMULA SHEET

Chapter 1 : Relations and Functions (NCERT)

1.1 Types of Relations

Relation	Condition (on set A)
Empty relation	No element related ; $R = \varphi$
Universal relation	Every element related ; $R = A \times A$
Reflexive	$(a, a) \in R$ for all $a \in A$
Symmetric	$(a, b) \in R \Rightarrow (b, a) \in R$
Transitive	$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
Equivalence relation	Reflexive + Symmetric + Transitive (all three)
Equivalence class [a]	Set of all elements related to a

1.2 Types of Functions

Function	Condition
One-one (injective)	$f(a) = f(b) \Rightarrow a = b$ (distinct inputs → distinct outputs)
Many-one	Two or more inputs give the same output
Onto (surjective)	Every element of codomain has a pre-image (range = codomain)
Into	At least one codomain element has no pre-image
Bijective	Both one-one AND onto
Number of bijections	From a set of n elements to itself = $n!$

1.3 Checking One-One & Onto

Method	Detail
One-one (algebraic)	Assume $f(x_1) = f(x_2)$ and prove $x_1 = x_2$
One-one (graphical)	Horizontal line cuts graph at most once
One-one (calculus)	$f'(x) > 0$ (strictly increasing) or $f'(x) < 0$ (strictly decreasing)
Onto	Show range = codomain (every y has some x)
Many-one example	$f(x) = x^2$ on R (since $f(-2) = f(2)$)

1.4 Composition of Functions

Concept	Formula
Composition	$(g \circ f)(x) = g(f(x))$
Order matters	$g \circ f \neq f \circ g$ (in general)
Associative	$(h \circ g) \circ f = h \circ (g \circ f)$
Composition of one-one	If f and g are one-one, then $g \circ f$ is one-one

Concept	Formula
Composition of onto	If f and g are onto, then $g \circ f$ is onto
Identity function	$I(x) = x$; $f \circ I = I \circ f = f$

1.5 Invertible Functions

Concept	Detail
Invertible function	A function f is invertible $\Leftrightarrow$ f is bijective (one-one & onto)
Inverse definition	$f^{-1}(y) = x \Leftrightarrow f(x) = y$
Property	$f \circ f^{-1} = f^{-1} \circ f$ = identity function
Inverse of composition	$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ (order reverses)
Finding inverse	Write $y = f(x)$, solve for x, then swap $\rightarrow f^{-1}(x)$
Inverse is unique	Each bijective function has exactly one inverse

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Equivalence relation	Must be Reflexive + Symmetric + Transitive — ALL three together.
One-one test	Set $f(x_1) = f(x_2)$; if it forces $x_1 = x_2$, the function is one-one.
Onto test	Range = codomain. Every output value must be achievable.
Bijective = invertible	A function has an inverse if and only if it is one-one AND onto.
Composition order	$(g \circ f)(x) = g(f(x))$ — apply f FIRST, then g. Order matters.
Inverse of composite	$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ — the order REVERSES (like socks & shoes).
Calculus shortcut	$f'(x)$ always >0 or always <0 $\rightarrow$ strictly monotonic $\rightarrow$ one-one.
Bijections count	Number of bijections from an n-set to itself = $n!$.

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Chapter 2 : Inverse Trigonometric Functions (NCERT)

2.1 Domains & Principal Value Ranges

Function	Domain	Principal Value Range
$\sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R (all reals)	$(-\pi/2, \pi/2)$
$\cot^{-1} x$	R (all reals)	$(0, \pi)$
$\sec^{-1} x$	$\lvert x\rvert \geq 1$	$[0, \pi] - \{\pi/2\}$
$\text{cosec}^{-1} x$	$\lvert x\rvert \geq 1$	$[-\pi/2, \pi/2] - \{0\}$

2.2 Basic Properties (Self-Inverse)

Property	Condition
$\sin(\sin^{-1} x) = x$	$x \in [-1, 1]$
$\sin^{-1}(\sin x) = x$	$x \in [-\pi/2, \pi/2]$
$\cos(\cos^{-1} x) = x$	$x \in [-1, 1]$
$\tan(\tan^{-1} x) = x$	$x \in R$
$\sin^{-1}(-x) = -\sin^{-1} x$	Odd function
$\tan^{-1}(-x) = -\tan^{-1} x$	Odd function
$\cos^{-1}(-x) = \pi - \cos^{-1} x$	Not odd
$\cot^{-1}(-x) = \pi - \cot^{-1} x$	Not odd

2.3 Complementary & Reciprocal Relations

Identity	Condition
$\sin^{-1} x + \cos^{-1} x = \pi/2$	$x \in [-1, 1]$
$\tan^{-1} x + \cot^{-1} x = \pi/2$	$x \in R$
$\sec^{-1} x + \text{cosec}^{-1} x = \pi/2$	$\lvert x\rvert \geq 1$
$\sin^{-1}(1/x) = \text{cosec}^{-1} x$	$\lvert x\rvert \geq 1$
$\cos^{-1}(1/x) = \sec^{-1} x$	$\lvert x\rvert \geq 1$
$\tan^{-1}(1/x) = \cot^{-1} x$	$x > 0$

2.4 Sum & Difference Formulas

Identity	Condition
$\tan^{-1} x + \tan^{-1} y = \tan^{-1}[(x+y)/(1-xy)]$	$xy < 1$
$\tan^{-1} x - \tan^{-1} y = \tan^{-1}[(x-y)/(1+xy)]$	$xy > -1$

Identity	Condition
$2\tan^{-1} x = \tan^{-1}[2x/(1-x^2)]$	$\|x\| < 1$
$2\tan^{-1} x = \sin^{-1}[2x/(1+x^2)]$	$\|x\| \le 1$
$2\tan^{-1} x = \cos^{-1}[(1-x^2)/(1+x^2)]$	$x \ge 0$
$2\sin^{-1} x = \sin^{-1}[2x\sqrt{(1-x^2)}]$	$-1/\sqrt{2} \le x \le 1/\sqrt{2}$

2.5 Useful Conversions ($\sin^{-1}$ in terms of others)

Identity	Equivalent Forms
$\sin^{-1} x =$	$\cos^{-1}\sqrt{(1-x^2)} = \tan^{-1}[x/\sqrt{(1-x^2)}]$
$\cos^{-1} x =$	$\sin^{-1}\sqrt{(1-x^2)} = \tan^{-1}[\sqrt{(1-x^2)}/x]$
$\tan^{-1} x =$	$\sin^{-1}[x/\sqrt{(1+x^2)}] = \cos^{-1}[1/\sqrt{(1+x^2)}]$
Useful identity	$3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$
Useful identity	$3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$
Useful identity	$3\tan^{-1} x = \tan^{-1}[(3x - x^3)/(1 - 3x^2)]$

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Principal ranges	$\sin^{-1}/\tan^{-1}/\text{cosec}^{-1} \to [-\pi/2,\pi/2]$ family ; $\cos^{-1}/\cot^{-1}/\sec^{-1} \to [0,\pi]$ family.
Complementary pairs	Each co-pair sums to $\pi/2$: $\sin^{-1}+\cos^{-1}$, $\tan^{-1}+\cot^{-1}$, $\sec^{-1}+\text{cosec}^{-1}$.
Odd functions	$\sin^{-1}$, $\tan^{-1}$, cosec^{-1} are odd: $f(-x) = -f(x)$.
$\cos^{-1}(-x)$	$= \pi - \cos^{-1}x$ (NOT odd). Same pattern for $\cot^{-1}$ and $\sec^{-1}$.
$\tan^{-1}$ sum	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}[(x+y)/(1-xy)]$ — watch the $xy<1$ condition.
Reciprocal rule	$\sin^{-1}(1/x) = \text{cosec}^{-1}x$, etc. Reciprocal inside ↔ co/reciprocal function.
Always check range	$\sin^{-1}(\sin x) = x$ ONLY if x is in the principal range; else adjust.
Triangle trick	For mixed conversions, draw a right triangle with the given ratio.

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Chapter 3 : Matrices (NCERT)

3.1 Basics & Order of a Matrix

Concept	Detail
Matrix	Rectangular array of numbers arranged in rows & columns
Order	$m \times n$ (m rows, n columns)
Element a_{ij}	Entry in i-th row, j-th column
Number of elements	$m \times n$
Equal matrices	Same order AND corresponding elements equal
Number of m×n matrices	Possible with given entries set

3.2 Types of Matrices

Type	Definition
Row matrix	Single row ($1 \times n$)
Column matrix	Single column ($m \times 1$)
Square matrix	Rows = columns ($n \times n$)
Diagonal matrix	Non-diagonal elements all zero
Scalar matrix	Diagonal matrix with equal diagonal elements
Identity matrix (I)	Scalar matrix with diagonal elements = 1
Zero matrix	All elements zero
Upper/lower triangular	Zeros below/above the main diagonal

3.3 Matrix Operations

Operation	Rule
Addition	Add corresponding elements (same order required)
Scalar multiplication	Multiply every element by the scalar k
Multiplication condition	$A(m\times n) \times B(n\times p)$ defined only if columns of A = rows of B
Product order	Result is ($m \times p$)
AB ≠ BA	Matrix multiplication is NOT commutative (in general)
Associative	$(AB)C = A(BC)$
Distributive	$A(B + C) = AB + AC$
Identity property	$AI = IA = A$

3.4 Transpose of a Matrix

Property	Formula
Transpose A'	Rows become columns (interchange)
(A')'	= A
(A + B)'	= A' + B'
(kA)'	= k A'
(AB)'	= B' A' (order reverses)
Symmetric matrix	A' = A ($a_{ij} = a_{ji}$)
Skew-symmetric matrix	A' = −A (diagonal elements = 0)

3.5 Symmetric & Skew-Symmetric

Concept	Detail
Symmetric	A' = A
Skew-symmetric	A' = −A ; main diagonal all zeros
Any square matrix	A = (symmetric part) + (skew-symmetric part)
Symmetric part	½(A + A')
Skew-symmetric part	½(A − A')
A + A'	Always symmetric
A − A'	Always skew-symmetric

3.6 Elementary Operations & Invertible Matrices

Concept	Detail
Elementary row operations	Swap rows, scale a row, add a multiple of one row to another
Invertible matrix	A is invertible if there exists B with AB = BA = I
Inverse notation	$B = A^{-1}$
Inverse uniqueness	Inverse of a matrix, if it exists, is unique
$(AB)^{-1}$	$= B^{-1} A^{-1}$ (order reverses)
Inverse by elementary ops	Convert A to I via row ops applied also to I
Condition to be invertible	Square matrix with non-zero determinant (covered in Ch 4)

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Multiplication rule	AB defined only when columns of A = rows of B. Result is rows(A) × cols(B).
Not commutative	AB ≠ BA in general — order always matters with matrices.
Transpose reverses	(AB)' = B'A' and $(AB)^{-1} = B^{-1}A^{-1}$ — order flips.
Symmetric vs skew	Symmetric: A' = A. Skew: A' = −A (diagonal all zeros).
Decomposition	Any square matrix = ½(A+A') + ½(A−A') (symmetric + skew parts).

Identity	AI = IA = A. The identity matrix acts like '1' for matrices.
Zero product trap	AB = 0 does NOT mean A = 0 or B = 0 (unlike numbers).
Inverse needs square	Only square matrices can have inverses ; and only if det ≠ 0.

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Chapter 4 : Determinants (NCERT)

4.1 Determinant — Definition

Concept	Formula
Determinant (2×2)	$|A| = a_{11}a_{22} - a_{12}a_{21}$
Determinant (3×3)	Expand along any row/column using cofactors
Notation	$\det A = |A| = \Delta$
Defined only for	Square matrices
Singular matrix	$|A| = 0$
Non-singular matrix	$|A| \neq 0$ (invertible)

4.2 Properties of Determinants

Property	Detail
Row–column swap	$|A'| = |A|$ (transpose leaves determinant unchanged)
Interchange two rows	Sign of determinant changes
Two identical rows/cols	Determinant = 0
Multiply a row by k	Determinant becomes $k|A|$
Common factor	k can be taken out of one row/column
Row addition	Adding a multiple of one row to another → no change
Triangular matrix	Determinant = product of diagonal elements
$|kA|$ (n×n)	$= k^n |A|$
$|AB|$	$= |A| \times |B|$

4.3 Area of Triangle & Minors/Cofactors

Concept	Formula
Area of triangle	$\Delta = \frac{1}{2} |x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$
Collinearity condition	Area = 0 (determinant of points = 0)
Minor M_{ij}	Determinant after deleting i-th row & j-th column
Cofactor A_{ij}	$A_{ij} = (-1)^{(i+j)} M_{ij}$
Expansion along a row	$|A| = a_{i1}A_{i1} + a_{i2}A_{i2} + a_{i3}A_{i3}$
Sum of products (wrong row)	= 0 (element × cofactor of another row)

4.4 Adjoint & Inverse

Concept	Formula
Adjoint	adj A = transpose of the cofactor matrix

Concept	Formula
A (adj A)	$= (\text{adj } A)\, A = \lvert A \rvert\, I$
Inverse	$A^{-1} = (\text{adj } A) \div \lvert A \rvert$, provided $\lvert A \rvert \neq 0$
|adj A| (n×n)	$= \lvert A \rvert^{(n-1)}$
adj(AB)	$= (\text{adj } B)(\text{adj } A)$
$\lvert A^{-1} \rvert$	$= 1 \div \lvert A \rvert$
$(A^{-1})^{-1}$	$= A$
A is invertible ⇔	$\lvert A \rvert \neq 0$ (non-singular)

4.5 Solving Linear Equations

Concept	Detail
Matrix form	$AX = B$ (A = coefficients, X = variables, B = constants)
Solution (matrix method)	$X = A^{-1} B$ (if $\lvert A \rvert \neq 0$)
Cramer's rule	$x = \Delta_1/\Delta$, $y = \Delta_2/\Delta$, $z = \Delta_3/\Delta$
Δ_i	Δ with i-th column replaced by constants B
Unique solution	If $\lvert A \rvert \neq 0$ (consistent, independent)
No solution	If $\lvert A \rvert = 0$ and $(\text{adj } A)B \neq 0$ (inconsistent)
Infinite solutions	If $\lvert A \rvert = 0$ and $(\text{adj } A)B = 0$ (dependent)

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2×2 determinant	$\lvert A \rvert = ad - bc$ (main diagonal minus off diagonal).
Zero determinant	Two equal rows/columns → det = 0. Such a matrix is singular (no inverse).
Row operations	Swapping rows flips sign ; adding a multiple of a row changes nothing.
Cofactor sign	$A_{ij} = (-1)^{(i+j)} M_{ij}$. Sign chessboard: + − + / − + − / + − +.
Inverse formula	$A^{-1} = \text{adj } A / \lvert A \rvert$. Needs $\lvert A \rvert \neq 0$.
|adj A|	$= \lvert A \rvert^{(n-1)}$ for an n×n matrix.
Cramer's rule	$x_i = \Delta_i/\Delta$ — replace the i-th column with constants.
Consistency	$\lvert A \rvert \neq 0$ unique ; $\lvert A \rvert = 0$ → either no solution or infinitely many.

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Chapter 5 : Continuity and Differentiability (NCERT)

5.1 Continuity

Concept	Detail
Continuous at x = a	LHL = RHL = f(a) ; i.e. $\lim_{x\to a} f(x) = f(a)$
Left-hand limit	$\lim_{x\to a^-} f(x)$
Right-hand limit	$\lim_{x\to a^+} f(x)$
Discontinuous if	Any of the three values differ or don't exist
Sum/diff/product	Continuous functions stay continuous under these
Quotient f/g	Continuous where $g \neq 0$
Composite	If f & g continuous, $g \circ f$ is continuous

5.2 Differentiability

Concept	Detail
Differentiable at x = a	LHD = RHD (left & right derivatives equal)
First principle	$f'(x) = \lim_{h\to 0}[f(x+h) - f(x)] \div h$
Key theorem	Differentiable ⇒ continuous (converse not always true)
Example (not differentiable)	$f(x) = \|x\|$ at $x = 0$ (continuous but sharp corner)
LHD	$\lim_{h\to 0^-}[f(a+h) - f(a)] \div h$
RHD	$\lim_{h\to 0^+}[f(a+h) - f(a)] \div h$

5.3 Standard Derivatives

Function	Derivative
x^n	$n\, x^{(n-1)}$
sin x / cos x	cos x / −sin x
tan x / cot x	$\sec^2 x$ / $-\text{cosec}^2 x$
sec x / cosec x	sec x tan x / −cosec x cot x
e^x / a^x	e^x / $a^x \log_e a$
$\log_e x$ / $\log_a x$	$1/x$ / $1/(x \log_e a)$
$\sin^{-1} x$ / $\cos^{-1} x$	$1/\sqrt{1-x^2}$ / $-1/\sqrt{1-x^2}$
$\tan^{-1} x$ / $\cot^{-1} x$	$1/(1+x^2)$ / $-1/(1+x^2)$
$\sec^{-1} x$ / $\text{cosec}^{-1} x$	$1/(\|x\|\sqrt{x^2-1})$ / $-1/(\|x\|\sqrt{x^2-1})$

5.4 Rules of Differentiation

Rule	Formula
Sum/difference	$(u \pm v)' = u' \pm v'$
Product rule	$(uv)' = u'v + uv'$
Quotient rule	$(u/v)' = (u'v - uv') \div v^2$
Chain rule	$d/dx\ f(g(x)) = f'(g(x)) \cdot g'(x)$
Implicit differentiation	Differentiate both sides w.r.t. x, solve for dy/dx
Parametric form	$dy/dx = (dy/dt) \div (dx/dt)$
Logarithmic differentiation	For $y = [f(x)]^{g(x)}$: take log, then differentiate

5.5 Special Differentiation & Second Derivative

Concept	Detail / Formula
$y = a^x$ form (use log)	$\ln y = x \ln a$; $(1/y)(dy/dx) = \ln a$
$y = x^x$	$dy/dx = x^{x}(1 + \ln x)$
Second derivative	$d^2y/dx^2 = d/dx\ (dy/dx)$
Parametric 2nd derivative	Differentiate dy/dx w.r.t. t, then divide by dx/dt
Higher derivatives	Differentiate repeatedly

5.6 Mean Value Theorems

Theorem	Statement
Rolle's theorem	If f continuous on [a,b], differentiable on (a,b), and f(a)=f(b), then f'(c)=0 for some c
Lagrange's MVT	$f'(c) = [f(b) - f(a)] \div (b - a)$ for some c in (a,b)
Geometric meaning (MVT)	Tangent at c is parallel to the chord joining endpoints
Conditions needed	Continuity on [a,b] and differentiability on (a,b)

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Continuity test	LHL = RHL = f(a). All three must match for continuity at a point.
Differentiable ⇒ continuous	True one way only. |x| is continuous at 0 but NOT differentiable.
Inverse trig derivatives	$\sin^{-1} \to 1/\sqrt{(1-x^2)}$; $\tan^{-1} \to 1/(1+x^2)$. Co-functions get a minus sign.
Chain rule	Differentiate outer, keep inner, times derivative of inner.
Log differentiation	Use for x^x, variable powers, and big products/quotients.
Parametric	dy/dx = (dy/dt)/(dx/dt). Don't forget to divide again for the 2nd derivative.
Rolle vs MVT	Rolle needs f(a)=f(b) and gives f'(c)=0 ; MVT is the general slope version.
x^x derivative	$d/dx(x^x) = x^x(1 + \ln x)$. A classic log-differentiation result.

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Chapter 6 : Application of Derivatives (NCERT)

6.1 Rate of Change

Concept	Formula
Rate of change	dy/dx = rate of change of y w.r.t. x
Related rates	Differentiate w.r.t. time t (chain rule)
Velocity	v = ds/dt
Acceleration	$a = dv/dt = d^2s/dt^2$
Marginal cost	MC = dC/dx (rate of change of cost)
Marginal revenue	MR = dR/dx

6.2 Increasing & Decreasing Functions

Concept	Condition
Increasing on interval	f'(x) > 0 for all x in the interval
Decreasing on interval	f'(x) < 0 for all x in the interval
Strictly increasing	f'(x) > 0 (no flat parts)
Strictly decreasing	f'(x) < 0
Constant function	f'(x) = 0 throughout
Critical points	Where f'(x) = 0 or f'(x) does not exist
Method	Find f'(x), set f'(x)=0, test sign in each interval

6.3 Tangents & Normals

Concept	Formula
Slope of tangent at (x_1,y_1)	m = (dy/dx) at that point
Equation of tangent	$y - y_1 = m(x - x_1)$
Slope of normal	−1/m (perpendicular to tangent)
Equation of normal	$y - y_1 = (-1/m)(x - x_1)$
Horizontal tangent	dy/dx = 0
Vertical tangent	dy/dx is undefined (dx/dy = 0)
Tangent parallel to x-axis	Slope = 0

6.4 Maxima & Minima

Concept	Detail
Critical points	Solve f'(x) = 0
First derivative test	f' changes + to − → maximum ; − to + → minimum

Concept	Detail
Second derivative test	f''(c) < 0 → maximum ; f''(c) > 0 → minimum
Point of inflection	f''(x) = 0 and sign of f'' changes
Local (relative) extrema	Highest/lowest in a neighbourhood
Absolute extrema	Compare values at critical points AND endpoints
If f''(c) = 0	Test fails ; use first derivative test

6.5 Working Rule for Absolute Max/Min

Step	Action
Step 1	Find f'(x) and solve f'(x) = 0 for critical points
Step 2	Evaluate f at all critical points in [a, b]
Step 3	Evaluate f at the endpoints a and b
Step 4	Largest value = absolute maximum ; smallest = absolute minimum
Optimisation problems	Form the function, differentiate, find the extremum

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Increasing/decreasing	f'(x) > 0 → increasing ; f'(x) < 0 → decreasing. Sign of the derivative tells direction.
Tangent & normal	Tangent slope = dy/dx ; normal slope = −1/(dy/dx) (perpendicular).
First derivative test	f' changes + → − = maximum ; − → + = minimum.
Second derivative test	f''(c) < 0 → max (cap) ; f''(c) > 0 → min (cup).
Test fails	If f''(c) = 0, fall back to the first derivative test.
Absolute extrema	Always check endpoints too, not just critical points.
Related rates	Differentiate w.r.t. time and link the rates via the chain rule.
Optimisation	Express the quantity as ONE variable, then maximise/minimise.

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Chapter 7 : Integrals (NCERT)

7.1 Standard Integrals (add + C)

Function	Integral
$\int x^n\,dx$	$x^{(n+1)}/(n+1)$, $n \neq -1$
$\int 1/x\,dx$	$\ln\lvert x\rvert$
$\int e^x\,dx$	e^x
$\int a^x\,dx$	$a^x / \ln a$
$\int \sin x\,dx$	$-\cos x$
$\int \cos x\,dx$	$\sin x$
$\int \sec^2 x\,dx$	$\tan x$
$\int \text{cosec}^2 x\,dx$	$-\cot x$
$\int \sec x \tan x\,dx$	$\sec x$
$\int \text{cosec}\, x \cot x\,dx$	$-\text{cosec}\, x$
$\int \tan x\,dx$	$\ln\lvert\sec x\rvert = -\ln\lvert\cos x\rvert$
$\int \cot x\,dx$	$\ln\lvert\sin x\rvert$
$\int \sec x\,dx$	$\ln\lvert\sec x + \tan x\rvert$
$\int \text{cosec}\, x\,dx$	$\ln\lvert\text{cosec}\, x - \cot x\rvert$

7.2 Special Integrals (Inverse Trig & Log Forms)

Integral	Result (+ C)
$\int 1/\sqrt{(1-x^2)}\,dx$	$\sin^{-1} x$
$\int 1/(1+x^2)\,dx$	$\tan^{-1} x$
$\int 1/(x\sqrt{(x^2-1)})\,dx$	$\sec^{-1} x$
$\int 1/(a^2 + x^2)\,dx$	$(1/a) \tan^{-1} (x/a)$
$\int 1/(x^2 - a^2)\,dx$	$(1/2a) \ln\lvert(x-a)/(x+a)\rvert$
$\int 1/(a^2 - x^2)\,dx$	$(1/2a) \ln\lvert(a+x)/(a-x)\rvert$
$\int 1/\sqrt{(a^2 - x^2)}\,dx$	$\sin^{-1} (x/a)$
$\int 1/\sqrt{(x^2 + a^2)}\,dx$	$\ln\lvert x + \sqrt{(x^2 + a^2)}\rvert$
$\int 1/\sqrt{(x^2 - a^2)}\,dx$	$\ln\lvert x + \sqrt{(x^2 - a^2)}\rvert$

7.3 Methods of Integration

Method	Detail
Substitution	Let $u = g(x)$; convert the integral in terms of u
$\int f(g(x))g'(x)\,dx$	$= \int f(u)\,du$ with $u = g(x)$

Method	Detail
Integration by parts	$\int u\,v\,dx = u\int v\,dx - \int (u' \int v\,dx)\,dx$
ILATE rule	Choose u in order: Inverse, Log, Algebraic, Trig, Exp
Partial fractions	Break a rational function into simpler fractions
$\int e^x[f(x)+f'(x)]\,dx$	$= e^x f(x) + C$

7.4 Special Forms (Integration by Parts results)

Integral	Result (+ C)
$\int \sqrt{(a^2 - x^2)}\,dx$	$(x/2)\sqrt{(a^2-x^2)} + (a^2/2)\sin^{-1}(x/a)$
$\int \sqrt{(x^2 + a^2)}\,dx$	$(x/2)\sqrt{(x^2+a^2)} + (a^2/2)\ln\lvert x+\sqrt{(x^2+a^2)}\rvert$
$\int \sqrt{(x^2 - a^2)}\,dx$	$(x/2)\sqrt{(x^2-a^2)} - (a^2/2)\ln\lvert x+\sqrt{(x^2-a^2)}\rvert$

7.5 Definite Integrals & Properties

Property	Formula
Fundamental theorem	$\int$(a to b) f(x) dx = F(b) − F(a) , F'(x)=f(x)
Reverse limits	$\int$(a to b) f = −$\int$(b to a) f
Same limits	$\int$(a to a) f = 0
King's property	$\int$(a to b) f(x) dx = $\int$(a to b) f(a+b−x) dx
Zero to a	$\int$(0 to a) f(x) dx = $\int$(0 to a) f(a−x) dx
Even function	$\int$(−a to a) f = 2$\int$(0 to a) f , if f even
Odd function	$\int$(−a to a) f = 0 , if f odd
Split interval	$\int$(a to b) = $\int$(a to c) + $\int$(c to b)

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Always + C	Indefinite integrals need the constant of integration C.
ILATE for parts	Pick u as Inverse > Log > Algebraic > Trig > Exp (in that priority).
e^x trick	$\int e^x[f(x)+f'(x)]dx = e^x f(x)$. Spot the function plus its derivative.
tan/sec integrals	$\int \tan x = \ln\lvert\sec x\rvert$; $\int \sec x = \ln\lvert\sec x + \tan x\rvert$ (memorise these).
$a^2 \pm x^2$ forms	$1/(a^2+x^2) \to (1/a)\tan^{-1}(x/a)$; $1/\sqrt{(a^2-x^2)} \to \sin^{-1}(x/a)$.
Partial fractions	Use when integrating proper rational functions (factor the denominator).
King's property	$\int_0^a f(x)dx = \int_0^a f(a-x)dx$ — powerful for definite integrals.
Even/odd	Over [−a,a]: even $\to 2\int_0^a$; odd $\to$ 0. Check symmetry first.

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Chapter 8 : Application of Integrals (NCERT)

8.1 Area Under a Curve

Concept	Formula
Area under curve $y = f(x)$	$A = \int(a \text{ to } b)\ y\,dx$ (between $x = a$ and $x = b$)
Area w.r.t. y-axis	$A = \int(c \text{ to } d)\ x\,dy$ (between $y = c$ and $y = d$)
Curve above x-axis	y is positive ; area $= \int y\,dx$
Curve below x-axis	y is negative ; take absolute value $\lvert\int y\,dx\rvert$
Area partly above & below	Split at x-intercepts ; add the absolute values

8.2 Area Between Two Curves

Concept	Formula
Between $y = f(x)$ and $y = g(x)$	$A = \int(a \text{ to } b)\ [f(x) - g(x)]\,dx$, $f \geq g$
Upper minus lower	Always (top curve − bottom curve)
Intersection points	Solve $f(x) = g(x)$ to get the limits a and b
Curves crossing	Split the interval where the curves cross
Between curves w.r.t. y	$A = \int(c \text{ to } d)\ [\text{right} - \text{left}]\,dy$

8.3 Standard Areas

Region	Area
Circle $x^2 + y^2 = a^2$	πa^2
Quarter circle (first quadrant)	$(1/4)\ \pi a^2$
Ellipse $x^2/a^2 + y^2/b^2 = 1$	$\pi a b$
Parabola $y^2 = 4ax$ (with a chord)	Use $\int x\,dy$ or $\int y\,dx$ as needed
Area of a triangle (vertices)	From coordinates via integration or determinant

8.4 Working Steps

Step	Action
Step 1	Draw a rough sketch of the curves/region
Step 2	Find intersection points (limits of integration)
Step 3	Decide strip direction (vertical dx or horizontal dy)
Step 4	Write area = $\int$ (top − bottom) dx (or right − left dy)
Step 5	Integrate and apply the limits ; take |value| if needed

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Basic area	Area under y = f(x) from a to b is ∫(a to b) y dx.
Below the axis	If the curve dips below the x-axis, the integral is negative — take |value|.
Between curves	Always integrate (TOP − BOTTOM). Find intersections for the limits first.
Sketch first	A rough sketch prevents sign errors and shows which curve is on top.
Symmetry	Use symmetry: e.g. circle area = 4 × (first-quadrant area).
Circle & ellipse	Circle: πa^2. Ellipse: πab. Handy checks for your answer.
dx vs dy	Pick horizontal strips (dy) when the region is simpler w.r.t. the y-axis.
Split regions	If curves cross inside the interval, split and add absolute areas.

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Chapter 9 : Differential Equations (NCERT)

9.1 Basic Concepts

Concept	Detail
Differential equation	Equation involving derivatives of a dependent variable
Order	Order of the highest derivative present
Degree	Power of the highest-order derivative (after removing radicals/fractions)
Degree condition	Defined only when equation is a polynomial in derivatives
Linear DE	Dependent variable & derivatives appear to first degree only
General solution	Contains arbitrary constants (= order of the DE)
Particular solution	Constants fixed by initial/boundary conditions

9.2 Formation of a Differential Equation

Step	Action
Step 1	Count the arbitrary constants in the given family
Step 2	Differentiate that many times
Step 3	Eliminate the constants using the equations
Result	DE order = number of arbitrary constants

9.3 Variable Separable Method

Concept	Formula
Separable form	$dy/dx = f(x)\, g(y)$
Separate variables	$dy/g(y) = f(x)\, dx$
Solution	$\int dy/g(y) = \int f(x)\, dx + C$
Simplest case	$dy/dx = f(x) \rightarrow y = \int f(x)\, dx + C$

9.4 Homogeneous Differential Equations

Concept	Detail
Homogeneous form	$dy/dx = F(y/x)$ (function of y/x only)
Substitution	Put $y = vx \Rightarrow dy/dx = v + x(dv/dx)$
After substitution	Becomes variable separable in v and x
Alternative (x as function of y)	Put $x = vy$ when easier
Then integrate	Separate and integrate, then replace $v = y/x$

9.5 Linear Differential Equations

Concept	Formula
Standard form (in y)	$dy/dx + P(x)\, y = Q(x)$
Integrating factor (IF)	IF = $e^{(\int P\, dx)}$
Solution	$y \cdot (IF) = \int Q \cdot (IF)\, dx + C$
Standard form (in x)	$dx/dy + P(y)\, x = Q(y)$
IF (for x form)	IF = $e^{(\int P\, dy)}$
Solution (x form)	$x \cdot (IF) = \int Q \cdot (IF)\, dy + C$

9.6 Quick Reference — Methods by Form

If the equation looks like…	Use this method
$dy/dx = f(x)\, g(y)$	Variable separable
$dy/dx = F(y/x)$	Homogeneous (put $y = vx$)
$dy/dx + Py = Q$	Linear (find IF = $e^{\int P\, dx)}$
$dx/dy + Px = Q$	Linear in x (find IF = $e^{\int P\, dy)}$
Just $dy/dx = f(x)$	Direct integration

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Order vs degree	Order = highest derivative ; degree = its power (only when polynomial in derivatives).
Constants = order	Number of arbitrary constants equals the order of the DE.
Separable	Get all y-terms with dy on one side, all x-terms with dx on the other, then integrate.
Homogeneous	$F(y/x)$ form → substitute $y = vx$, then it becomes separable.
Linear IF	IF = $e^{(\int P\, dx)}$. Solution: $y \cdot IF = \int Q \cdot IF\, dx + C$.
Spot the form	Identify separable / homogeneous / linear FIRST — that picks the method.
x as variable	If P, Q depend on y, write $dx/dy + Px = Q$ and use IF in y.
Always + C	General solution always carries the arbitrary constant(s).

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Chapter 10 : Vector Algebra (NCERT)

10.1 Basics & Types of Vectors

Concept	Detail
Vector	Quantity with magnitude AND direction
Magnitude of a = xî + yĵ + zk̂	$\|a\| = \sqrt{(x^2 + y^2 + z^2)}$
Position vector of P(x,y,z)	r = xî + yĵ + zk̂
Zero vector	Magnitude 0, no fixed direction
Unit vector	$\hat{a} = a / \|a\|$ (magnitude 1)
Equal vectors	Same magnitude AND same direction
Collinear vectors	Parallel ; a = λb for some scalar λ
Coplanar vectors	Lie in the same plane

10.2 Vector Operations & Section Formula

Concept	Formula
Addition (triangle/parallelogram)	Resultant of two vectors
Scalar multiplication	λa scales magnitude by $\|\lambda\|$
Direction cosines	$l = x/\|r\|$, $m = y/\|r\|$, $n = z/\|r\|$
Property of DCs	$l^2 + m^2 + n^2 = 1$
Section formula (internal)	r = (m b + n a) ÷ (m + n)
Section formula (external)	r = (m b − n a) ÷ (m − n)
Midpoint	r = (a + b) ÷ 2

10.3 Dot (Scalar) Product

Concept	Formula
Definition	$a \cdot b = \|a\|\|b\| \cos \theta$
Component form	$a \cdot b = a_1b_1 + a_2b_2 + a_3b_3$
Angle between vectors	$\cos \theta = (a \cdot b) \div (\|a\|\|b\|)$
Perpendicular vectors	$a \cdot b = 0$
Magnitude squared	$a \cdot a = \|a\|^2$
Projection of a on b	$(a \cdot b) \div \|b\|$
Commutative	$a \cdot b = b \cdot a$
Result type	Scalar (a number)

10.4 Cross (Vector) Product

Concept	Formula
Definition	$a \times b = |a||b| \sin \theta\ \hat{n}$
Magnitude	$|a \times b| = |a||b| \sin \theta$
Component form	$a \times b$ = determinant of $[\hat{i}\ \hat{j}\ \hat{k}\ ;\ a_1\ a_2\ a_3\ ;\ b_1\ b_2\ b_3]$
Parallel vectors	$a \times b = 0$
Anti-commutative	$a \times b = -(b \times a)$
Area of parallelogram	$|a \times b|$ (adjacent sides a, b)
Area of triangle	$\frac{1}{2} |a \times b|$
Unit normal vector	$\hat{n} = (a \times b) \div |a \times b|$
Result type	Vector (perpendicular to both)

10.5 Dot vs Cross — Comparison

Feature	Dot Product	Cross Product
Result	Scalar	Vector
Formula	$|a||b| \cos \theta$	$|a||b| \sin \theta\ \hat{n}$
Zero when	Perpendicular ($\theta = 90°$)	Parallel ($\theta = 0°$)
Commutative?	Yes ($a \cdot b = b \cdot a$)	No ($a \times b = -b \times a$)
Main use	Angle, projection	Area, normal direction

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Magnitude	$|a| = \sqrt{x^2+y^2+z^2}$. Unit vector = $a/|a|$.
Dot = scalar	$a \cdot b = |a||b|\cos\theta$. Zero when perpendicular. Gives the angle & projection.
Cross = vector	$a \times b = |a||b|\sin\theta\ \hat{n}$. Zero when parallel. Gives area & normal direction.
cos vs sin	Dot uses COSine (cos→perpendicular zero) ; cross uses SINe (sin→parallel zero).
Areas	Parallelogram = $|a \times b|$; triangle = $\frac{1}{2}|a \times b|$.
Direction cosines	$l^2+m^2+n^2 = 1$ always.
Section formula	Internal: $(mb+na)/(m+n)$. Midpoint: $(a+b)/2$.
Cross order matters	$a \times b = -(b \times a)$. Dot order doesn't: $a \cdot b = b \cdot a$.

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Chapter 11 : Three Dimensional Geometry (NCERT)

11.1 Direction Cosines & Direction Ratios

Concept	Formula
Direction cosines (DCs)	$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$
Property of DCs	$l^2 + m^2 + n^2 = 1$
Direction ratios (DRs)	a, b, c proportional to l, m, n
DCs from DRs	$l = a/\sqrt{(a^2+b^2+c^2)}$, and similarly for m, n
DRs from two points	$(x_2-x_1, y_2-y_1, z_2-z_1)$
Distance between points	$\sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2]}$

11.2 Equation of a Line

Form	Equation
Vector form (point + direction)	$r = a + \lambda b$
Cartesian (symmetric) form	$(x-x_1)/a = (y-y_1)/b = (z-z_1)/c$
Through two points (vector)	$r = a + \lambda(b - a)$
Through two points (Cartesian)	$(x-x_1)/(x_2-x_1) = (y-y_1)/(y_2-y_1) = (z-z_1)/(z_2-z_1)$
a, b, c here are	Direction ratios of the line

11.3 Angle Between Two Lines

Concept	Formula
Angle (direction ratios)	$\cos\theta = \lvert a_1a_2 + b_1b_2 + c_1c_2\rvert \div (\sqrt{(a_1^2+b_1^2+c_1^2)}\ \sqrt{(a_2^2+b_2^2+c_2^2)})$
Angle (vector form)	$\cos\theta = \lvert b_1 \cdot b_2\rvert \div (\lvert b_1\rvert\lvert b_2\rvert)$
Parallel lines	$a_1/a_2 = b_1/b_2 = c_1/c_2$
Perpendicular lines	$a_1a_2 + b_1b_2 + c_1c_2 = 0$

11.4 Equation of a Plane

Form	Equation
General Cartesian form	$Ax + By + Cz + D = 0$
Normal vector to plane	(A, B, C) are direction ratios of the normal
Vector form (normal n)	$r \cdot \hat{n} = d$ (d = distance from origin)
Point + normal form	$A(x-x_1) + B(y-y_1) + C(z-z_1) = 0$
Intercept form	$x/a + y/b + z/c = 1$
Plane through 3 points	Use determinant of coordinates = 0

11.5 Angles & Distances

Concept	Formula
Angle between two planes	$\cos\theta = \lvert A_1A_2+B_1B_2+C_1C_2\rvert \div (\sqrt{(\Sigma A_1^2)}\ \sqrt{(\Sigma A_2^2)})$
Parallel planes	$A_1/A_2 = B_1/B_2 = C_1/C_2$
Perpendicular planes	$A_1A_2 + B_1B_2 + C_1C_2 = 0$
Angle between line & plane	$\sin\theta = \lvert a\cdot n\rvert \div (\lvert a\rvert\lvert n\rvert)$
Distance: point to plane	$\lvert Ax_1+By_1+Cz_1+D\rvert \div \sqrt{(A^2+B^2+C^2)}$
Line parallel to plane	$a\cdot n = 0$ (direction ⊥ normal)

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Direction cosines	$l^2+m^2+n^2 = 1$ always. DCs are the cosines of angles with the axes.
Line equation	Vector: $r = a + \lambda b$ (point a, direction b). Cartesian uses the same DRs.
Parallel vs perpendicular	Parallel: DRs proportional. Perpendicular: $a_1a_2+b_1b_2+c_1c_2 = 0$.
Plane normal	In $Ax+By+Cz+D=0$, the coefficients (A,B,C) form the normal vector.
Angle line–plane	Uses SINE ($\sin\theta = \lvert a\cdot n\rvert/\lvert a\rvert\lvert n\rvert$) — note: with the NORMAL.
Angle line–line / plane–plane	Uses COSINE of the direction/normal vectors.
Point–plane distance	$\lvert Ax_1+By_1+Cz_1+D\rvert / \sqrt{(A^2+B^2+C^2)}$. Plug the point into the plane.
Intercept form	$x/a + y/b + z/c = 1$ gives axis intercepts a, b, c directly.

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Chapter 12 : Linear Programming (NCERT)

12.1 Basic Terms

Term	Meaning
Linear programming problem (LPP)	Optimising (max/min) a linear function under linear constraints
Objective function	$Z = ax + by$ (the quantity to maximise/minimise)
Decision variables	x, y (the unknowns to be found)
Constraints	Linear inequalities restricting x, y
Non-negative restrictions	$x \geq 0$, $y \geq 0$
Optimal value	The maximum or minimum value of Z

12.2 Feasible Region & Solutions

Concept	Meaning
Feasible region	Region satisfying ALL constraints (incl. $x,y \geq 0$)
Feasible solution	Any point inside or on the feasible region
Infeasible region	Region not satisfying the constraints
Bounded region	Can be enclosed in a circle (has finite extent)
Unbounded region	Extends infinitely in some direction
Corner point (vertex)	Point where constraint boundaries intersect
Optimal solution	The corner point giving the best Z value

12.3 Corner Point Method (Working Rule)

Step	Action
Step 1	Formulate the LPP (objective function + constraints)
Step 2	Plot the constraints and shade the feasible region
Step 3	Find all corner points (vertices) of the feasible region
Step 4	Evaluate $Z = ax + by$ at each corner point
Step 5 (bounded)	Largest value = maximum, smallest = minimum
Step 6 (unbounded)	Check if optimum exists using open half-plane test

12.4 Unbounded Region — Special Checks

Case	Rule
Maximum (unbounded)	If $ax + by > M$ has points in feasible region, no max exists
Minimum (unbounded)	If $ax + by < m$ has points in feasible region, no min exists
Otherwise	The value M (or m) is the optimal value

Case	Rule
Multiple optimal solutions	If two corner points give the same optimal Z, every point on that edge is optimal

12.5 Types of LPP

Type	Goal
Manufacturing problem	Maximise profit / production
Diet problem	Minimise cost while meeting nutrient needs
Transportation problem	Minimise the cost of transporting goods
Allocation problem	Best allocation of limited resources

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Objective function	$Z = ax + by$ — the linear quantity you maximise or minimise.
Feasible region	Satisfies ALL constraints, including $x \geq 0$ and $y \geq 0$.
Optimum at corners	The max/min of Z always occurs at a CORNER point of the feasible region.
Corner point method	Evaluate Z at every vertex ; pick the largest (max) or smallest (min).
Bounded region	A bounded feasible region always has both a max and a min.
Unbounded check	For unbounded regions, test the open half-plane to confirm the optimum exists.
Multiple solutions	Equal Z at two corners → every point on that edge is optimal.
Always sketch	Plot constraints carefully ; the feasible region drives the whole answer.

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Chapter 13 : Probability (NCERT)

13.1 Conditional Probability

Concept	Formula
Conditional probability	$P(A\|B) = P(A \cap B) \div P(B)$, $P(B) \neq 0$
Multiplication theorem	$P(A \cap B) = P(B)\, P(A\|B) = P(A)\, P(B\|A)$
For three events	$P(A \cap B \cap C) = P(A)\, P(B\|A)\, P(C\|A \cap B)$
Range	$0 \leq P(A\|B) \leq 1$
P(S|B)	$= 1$
Complement	$P(A'\|B) = 1 - P(A\|B)$

13.2 Independent Events

Concept	Condition
Independent events	$P(A \cap B) = P(A) \times P(B)$
Then P(A|B)	$= P(A)$ (B does not affect A)
Three independent events	$P(A \cap B \cap C) = P(A)P(B)P(C)$
Independent vs mutually exclusive	Different ideas — don't confuse them
Mutually exclusive	$P(A \cap B) = 0$ (cannot occur together)
If A, B independent	A & B', A' & B, A' & B' are also independent

13.3 Total Probability & Bayes' Theorem

Concept	Formula
Total probability theorem	$P(A) = \Sigma\, P(E_i)\, P(A\|E_i)$
Partition condition	$E_1, E_2, \ldots$ are mutually exclusive & exhaustive
Bayes' theorem	$P(E_i\|A) = [P(E_i)P(A\|E_i)] \div [\Sigma\, P(E_j)P(A\|E_j)]$
Prior probability	$P(E_i)$ — before the event A is known
Posterior probability	$P(E_i\|A)$ — revised after A occurs
Use of Bayes	Find the cause given the observed effect

13.4 Random Variable & Probability Distribution

Concept	Formula
Random variable (X)	Real-valued function on the sample space
Probability distribution	Lists X values with their probabilities p_i
Condition	$\Sigma\, p_i = 1$ (all probabilities sum to 1)
Mean (expectation)	$E(X) = \mu = \Sigma\, x_i\, p_i$

Concept	Formula
Variance	$Var(X) = \Sigma x_i^2 p_i - \mu^2$
Standard deviation	$\sigma = \sqrt{(Var(X))}$

13.5 Bernoulli Trials & Binomial Distribution

Concept	Formula
Bernoulli trial	Trial with two outcomes: success / failure
Conditions	Fixed n trials, independent, constant p
Binomial probability	$P(X = r) = {}^nC_r \, p^r \, q^{(n-r)}$
q (failure)	$q = 1 - p$
Mean of binomial	$\mu = n p$
Variance of binomial	$\sigma^2 = n p q$
Standard deviation	$\sigma = \sqrt{(n p q)}$
Note	Mean > variance (since $q < 1$)

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Conditional	$P(A\|B) = P(A \cap B)/P(B)$. Multiplication: $P(A \cap B) = P(A)P(B\|A)$.
Independent	$P(A \cap B) = P(A)P(B)$. Independent ≠ mutually exclusive!
Mutually exclusive	$P(A \cap B) = 0$ (can't both happen) — opposite idea to independence.
Total probability	$P(A) = \Sigma P(E_i)P(A\|E_i)$ over a partition of the sample space.
Bayes' theorem	Reverses the condition: find P(cause|effect) from P(effect|cause).
Distribution check	All probabilities must sum to 1 ($\Sigma p_i = 1$).
Mean & variance	$E(X) = \Sigma x_i p_i$; $Var(X) = \Sigma x_i^2 p_i - \mu^2$.
Binomial	$P(X=r) = {}^nC_r \, p^r q^{(n-r)}$; mean = np, variance = npq.